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Adaptive Damping of Vibration by the Prestress Accumulation/Release Strategy

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7.1 Introduction

The majority of the recently discussed active damping of vibration techniques in engineering structures are based on magnetorheological fluids (semi-active and applicable to real structures; e.g. see Reference [1]) and piezoelectric devices (fully active, but applicable mostly to lab-scale demonstrators, e.g. see Reference [2]).

The prestress accumulation/release (PAR) is a method to convert the strain energy of a vibrating system into kinetic energy, which is then released from the system by means of a dissipative device. The process is fully semi-active; it does not require adding any substantial amount of energy into the system. As a consequence, it creates the possibility of applications to real structures. The semi-active methods are popular because of their high efficiency and relatively low cost, compared to passive and active damping of vibrations [3,4]. Some interesting investigations of semi-active techniques have been published in References [5] and [6].

The first formulation of the PAR concept for a truss-beam system and double-layered cantilever beam was presented in References [7] and [8]. Based on the use of an on-off joint connections control concept for energy dissipation in a flexible truss-beam structure, a similar formulation was presented in Reference [9], whereas Reference [10] investigates the vibration suppression in a mass-two-spring system due to controlled detaching and reattaching of a spring.

The dissipating process discussed in the present chapter consists of two phases. In the first phase, some kinematic constraints imposed on the system are released at the instant when the maximum strain energy can be converted to kinetic energy. It is usually manifested by local, high-frequency vibrations. In the second phase, kinematic constraints are reimposed, which leads to conversion of a part of the kinetic energy into another, nonmechanical form, e.g. heating-up of the actuator device.

First, the proposed approach is described theoretically on a simple spring–mass system in order to demonstrate the idea of response mitigation and to show the energy balance of the system. Second, the numerical studies are presented for a layered beam simulating a pedestrian bridge, where the control is based on disconnecting (for a very short instant of time) and then sticking back two layers (delamination effect).

Finally, the experimental results are presented. A laboratory-scale set-up was built to verify the effectiveness of the PAR strategy on a cantilever beam demonstrator. A controllable delamination effect was obtained by means of piezoelectric actuators. The control was carried out as a closed-loop feedback system.

7.2 Mass–Spring System

7.2.1 The Concept

A simple ‘mass–two-spring’ system is considered, as shown in Figure 7.1(a). One of the springs is active in the sense that it can be detached and then reattached to the mass anywhere along the spring length. During the free vibration of the system, the active spring can be detached, in particular at the point of maximum displacement of the mass, and reattached as it comes to its free end (cf. Figure 7.1(b)). In the following phase of vibration, a force that opposes further movement is introduced, proportional to the displacement of the active spring from its new equilibrium position. Thus, a new equilibrium of the whole system is established (dotted line in Figure 7.1(c)). Then the active spring can be detached and reattached again, resulting in returning to the initial configuration. During the following vibration, the system behaves the same as before introducing the control, but a considerable part of the total vibration amplitude vanishes.

7.2.2 Analytical Solution

7.2.2.1 Equation of Motion

In this section the system shown in Figure 7.1 is analysed in detail. If natural damping is not considered and no force excitation is used, then the motion in the first phase of vibration is governed by the equation:

$$m \ddot{x}(t) + (k_1 + k_2) x(t) = 0 \quad (1)$$

where m is the moving mass and $k_i = E_i A_i / L_i$ is the stiffness of a spring with cross-section A_i , Young’s modulus E_i and length L_i . The solution, under the given initial conditions

Figure 7.1 Mass–two-spring system

$x(t = 0) = -\epsilon$ and $\dot{x}(t = 0) = 0$, takes the form:

$$x(t) = -\epsilon \cos \left(\sqrt{\frac{k_1 + k_2}{m}} t \right) \quad (2)$$

At the time instant of maximum displacement, $t = t^1$, the active spring is detached and reattached as it comes to its equilibrium position (cf. Figure 7.1(b)). At this point it is assumed that the inertia of springs is not taken into account. Now the equation of motion is given by

$$m \ddot{x}(t) + k_1 x(t) + k'_2 x_2(t) = 0 \quad (3)$$

where $k'_2 = E_2 A_2 / L'_2$ with new active spring length $L'_2 = L_2 - \epsilon$ and x_2 is the active spring displacement in the second phase: $x_2(t) = x(t) - \epsilon$. The system has still a single degree of freedom, namely the displacement of the mass m . Equation (3) can be rewritten as

$$m \ddot{x}(t) + (k_1 + k'_2) x(t) = k'_2 \epsilon \quad (4)$$

It can be seen that in the second phase of the process, the governing equation is nonhomogeneous with a term $k'_2 \epsilon$, which can be viewed as an additional constant force applied to the system. Now the solution takes the form:

$$x(t) = C_1 \cos \left(\sqrt{\frac{k_1 + k'_2}{m}} t \right) + C_2 \sin \left(\sqrt{\frac{k_1 + k'_2}{m}} t \right) + \frac{k'_2 \epsilon}{k_1 + k'_2} \quad (5)$$

with constants C_1 and C_2 calculated from the initial conditions: $x(t = t^1) = \epsilon$ and $\dot{x}(t = t^1) = 0$. This operation can be repeated several times in order to enhance the amplitude mitigation effect. If the desired effectiveness is reached, then the operation of 'detaching/reattaching' can be repeated again near the initial equilibrium position, in order for the active spring to return to its initial length.

7.2.2.2 Energy Balance

The potential energy of a spring is equal to the work done by the elastic force along the displacement direction:

$$E_{\text{pot}} = \int_0^{x_k} k_i x \, dx = \frac{1}{2} k_i x_k^2 \quad (6)$$

The potential energy of the system at the instant before activation of the control $t = t^1 - dt$ is

$$E_{\text{pot}} = \frac{1}{2} k_1 \epsilon^2 + \frac{1}{2} k_2 \epsilon^2 \quad (7)$$

The potential energy of the system at the instant after activation of control $t = t^1 + dt$ is

$$E_{\text{pot}} = \frac{1}{2} k_1 \epsilon^2 + \frac{1}{2} k'_2 (\epsilon - \epsilon)^2 = \frac{1}{2} k_1 \epsilon^2 \quad (8)$$

The control is activated at the point of maximum displacement, where $\dot{x} = 0$, thus at this point the kinetic energy vanishes. If the inertia of springs was not neglected then some energy would

Figure 7.2 Resulting mass displacement; the control is triggered twice

be transferred to this part of the active spring, which can vibrate freely after reattachment. This situation will be discussed in further sections.

7.2.2.3 Numerical Example

Figure 7.2 depicts the resulting displacement of the mass if the following data were used for calculations (both springs equal):

$$m = 20 \text{ kg}, \quad L = 0.1 \text{ m}, \quad E = 6 \times 10^{10} \text{ Pa}, \quad A = 1.54 \times 10^{-8} \text{ m}^2, \quad \epsilon = 0.01 \text{ m}$$

After performing detaching/reattaching of the active spring twice, vibrations of the mass have ceased almost completely. A slight change in frequency of the controlled response, as compared with the reference case, is due to a small change in the spring stiffness, which affects the frequency and is caused by a change in the spring length. It can also be observed that after activating the control, the system oscillates about the new equilibrium position (cf. Figure 7.2).

Activating control takes place at the maximum value of the accumulated strain energy and results in an instant decrease in the strain energy (cf. Figure 7.3 and Equations (7) and (8)). At

Figure 7.3 Energy balance

the end of the process, almost all the energy is released from the system. In a more practical approach, part of the energy would be transferred into higher frequency vibrations of the detached part of the active spring and another part would be dissipated in the process, which is here idealized as imposing some initial values. The last phase of the process is shown in Figure 7.3(b).

7.2.3 Case with Inertia of the Active Spring Considered

7.2.3.1 Introduction

As stated before, detaching the active spring results in converting the accumulated strain energy into kinetic energy, which can be dissipated from the system during reattachment of the spring. This whole process was idealized in the previous section by imposing proper initial conditions, which resulted in an instant decrease in energy of the system. In practice, part of the released strain energy is dissipated by a device that reattaches the spring and the remaining part introduces higher frequency vibrations, which can be easily suppressed by natural damping of the system.

In the present analysis, the control device is idealized by imposing/releasing local constraints between the geometrical point of mass m and any point along the active spring. The mass of active spring is concentrated at its full length and in the middle (cf. Figure 7.4). Slight natural damping is also introduced into the equations. The Rayleigh damping coefficients introduce little damping (1 or 0.2 %) around the first natural frequency and relatively much higher damping of higher frequencies. All remaining parameters do not change. (Simulations were performed using the Abaqus/Standard code.)

7.2.3.2 Results

Displacements of mass m and the tip of the spring are depicted in Figure 7.5. Detaching an active spring and reattaching it at some point along its length introduces higher frequency vibrations of the spring loose end. Releasing/reimposing the constraint again results in returning the system to the initial configuration, but with the amplitude of vibrations decreased.

First, the spring is detached at the point of maximum displacement, i.e. where there is the maximum stress accumulated in the spring. Then the spring is reattached when its end passes the equilibrium position, which means that the length of the spring is decreased by about 10 mm. This corresponds to Figure 7.4(b). If the control is stopped there, the system would oscillate about a new equilibrium position. The time instant for the next spring detachment has to be chosen properly, so that it can be reattached as the mass m is as close to the initial equilibrium position as possible. The whole procedure can be repeated several times, as needed. If, for instance, the active spring stiffness is considerably smaller compared to the passive one, then the desired mitigation effect would have to be obtained in more steps. In the analysed

Figure 7.4 Dynamic model with inertia of the active spring included

Figure 7.5 Response of the system: (a) 1 % of critical damping, (b) 0.2 % of critical damping

example, after the described sequence of activating the control, the amplitude of the mass m displacement was decreased by 96 % and 92 % for 1 % and 0.2 % of critical damping, respectively. The procedure is very sensitive to the time instant of detaching the spring.

Reimposing the constraints causes higher frequency vibrations of the mass located in the middle of the spring. These vibrations, however, are effectively damped out by the natural damping of the system. The typical behavior of the middle mass is shown in Figure 7.6.

Figure 7.6 Middle mass response with indicated points corresponding to the activating control

Figure 7.7 (a) Sum of strain and kinetic energy, (b) viscous dissipation due to natural damping, (c) loss of kinetic energy while imposing constraints

Steep exponential decline in the sum of a potential and kinetic energy graph is due to viscous dissipation, which increases with an increase in the vibration velocity. Vibration velocity, in turn, is at its highest when the local, higher frequency vibrations are introduced due to imposing/releasing the constraints. The viscous dissipation is due to natural damping of the system.

Discontinuity of the graph is caused by the loss of kinetic energy at the instant when constraints are reimposed. The size of this gap indicates the maximum amount of energy that can be dissipated by the active device. All nonzero forms of energy can be seen in Figure 7.7.

7.3 Delamination of a Layered Beam

7.3.1 PAR Strategy for Layered Beams

The strategy of releasing the accumulated strain energy in order to dissipate it can in theory be effectively used for various types of structures. If a layered beam is considered as shown

Figure 7.8 PAR strategy for a cantilever beam

in Figure 7.8, the idea of adaptation would be as follows. First, at the point of maximum deflection two layers are disconnected, resulting in almost instant dislocation of layers (1' in Figure 7.8(a)). This dislocation can then be frozen if the layers are reconnected again. This yields an introduction of the elastic force that opposes further vibration of the cantilever (2 in Figure 7.8(a)). Then, near the equilibrium position the layers are disconnected/reconnected again in order to return to the initial configuration. The whole sequence can be repeated until the desired effect is obtained.

A similar effect of response mitigation can be obtained if a truss structure is considered with a detachable element (cf. Figure 7.8(b)). Applying the same methodology for control, the axial strain accumulated in the active element can be released as the element is disconnected at one of its ends. It is worth mentioning that in both cases only one active member is required.

7.3.2 Numerical Example of a Simply Supported Beam

7.3.2.1 Numerical Model

A simply supported, two-layered beam with the span of 15.6 m is considered. The bending stiffness of each layer is $EI = 2.218 \times 10^6 \text{ N m}^2$. Material damping of 1% of critical damping is assumed around the first natural frequency. Layers are permanently connected together at the left support. It is assumed that there is a device at the right support, capable of instantly disconnecting or sticking the layers. Along the beam length the distance between the layers remains the same, whereas the frictionless, relative movement of layers is possible in the direction parallel to the beam axis. The considered beam model is depicted in Figure 7.9.

Figure 7.9 Assumed model of the layered beam

Figure 7.10 Vertical displacement of the middle of the beam

7.3.2.2 Results

In the first step, an initial displacement of 16 cm was applied to the model. In the following steps, the dynamic analysis procedure was used to calculate the free vibrations of the system as in the reference case. Then the calculations were repeated with control procedures added. The vertical displacement of the middle of the beam is shown in Figure 7.10.

It can be seen that ca. 95 % of the vibration amplitude is damped out after two cycles of vibration. The control signal, i.e. the relative displacement between layer ends shown in the picture, is magnified 10 times. After only two cycles of disconnection/reconnection of layers, the vibration of the first mode is considerably mitigated, while higher modes of vibrations are introduced. The second sequence of control activation is triggered close to the maximum accumulated strain, at the point where the deformation shape is of the first mode (cf. Figure 7.11(a)). It is worth mentioning that if the control is triggered at the local peak, shortly after the first sequence, it would not give the desired effect, since the deformation shape at that point is of the third mode of vibration (cf. Figure 7.11(b)). The performed control does not affect higher modes, which can, however, be mitigated by the natural damping. Appropriate modification of the control strategy could also make PAR a useful tool in higher mode mitigation.

The control in the numerical simulations was applied by instant releasing of the available component of relative motion between the layers or freezing it at some value. This resulted in a very fast dislocation of layers in the direction parallel to the beam axis. Of course, a drawback to such an approach is that between the subsequent disconnecting and reconnecting of layers, the beam stiffness is decreased. However, the time intervals when this is the case are very short,

Figure 7.11 Desired (a) and undesired (b) deformation shapes for activating the control

Figure 7.12 Relative displacement of layers

as can be seen in Figure 7.12. It can also be observed that at the end of the process, relative displacement between the layers is zero, which means that the structure has returned to the initial configuration.

7.3.3 PAR – the VDM Formulation

The VDM-based model of a composite beam with the contact layer presented in Chapter 3 (Section 3.3.2) can be effectively used to describe the PAR process. Assuming feasibility of the full delamination/adhesion actions (performed by smart actuators), the following formula can be used to determine strains in the diagonal truss elements A and B (cf. Figure 3.16) of the contact layer:

$$\varepsilon_i(t > t_1) = \varepsilon_i^L(t) + \sum_{\tau=0}^t D_{ij}(t - t_1 - \tau) \varepsilon_j^0(\tau) \quad (9)$$

where ε_i^L denotes the response of the fully bonded two-layer beam, t_1 denotes the time instant of introducing delamination (readhesion is taking place immediately afterwards at $t_1 + \delta t$) and ε_j^0 is constant and equal to the strain in the diagonal element A (or B) at the time instant t_1 . The virtual distortions introduced at the time instant t_1 have to be removed at the time instant t_2 , when the maximum rebound of the two bonded (by actuators) layers is taking place. This means that an extra component has to be added to the strain formula:

$$\varepsilon_i(t) = \varepsilon_i^L(t) + \sum_{\tau=0}^t D_{ij}(t - t_1 - \tau) \varepsilon_j^0(\tau) - \sum_{\tau=0}^t D_{ij}(t - t_2 - \tau) \varepsilon_j^0(\tau) \quad (10)$$

where ε_j^0 is constant over the time period $\langle t_1, t_2 \rangle$. The formula (10) specifies the strains related to the accumulated and released parts of the energy.

If the damping process is less effective, the delamination/adhesion actions have to be repeated several times and the corresponding virtual distortions, modeling the prestress accumulation and release at the contact interface, can be written as follows:

$$\varepsilon_i(t) = \varepsilon_i^L(t) + \sum_{\tau=0}^t \sum_{\alpha} D_{ij}(t - t_1^{\alpha} - \tau) \varepsilon_j^0(\tau, \alpha) - \sum_{\tau=0}^t \sum_{\alpha} D_{ij}(t - t_2^{\alpha} - \tau) \varepsilon_j^0(\tau, \alpha) \quad (11)$$

where α denotes the number of successive cycles of accumulating and releasing the energy in the time intervals $\langle t_1^{\alpha}, t_2^{\alpha} \rangle$. Each cycle (characterized by constant distortion ε_j^0) starts with activation of the actuator (adhesion) and finishes with its deactivation (delamination), see Figure 7.13). Note that the cycles occur continuously one after another, i.e. $t_2^{\alpha} = t_1^{\alpha+1}$. Thus, the algorithm of the PAR strategy simulation consists of the following stages:

1. Initialize – influence the matrix, linear response of the structure.
2. For every time step:
 - track the instant in which the potential energy of the structure is maximal;
 - update the α value and take a new $\varepsilon_j^0(\tau, \alpha)$;
 - solve the set (11) for the structural response.

Figure 7.13 (a) Double-layer beam model, (b) time history of the strain collected by the sensor in the case of a single delamination/adhesion action, (c) time history of the strain collected by the sensor in the case of a few delamination/adhesion actions

The presented algorithm was applied to a double-layer cantilever beam (the material properties of the structural elements were $E = 80 \times 10^9$ Pa, $A = 1 \times 10^{-5}$ m²). The structure was vibrating at the fourth eigenfrequency. The virtual distortions were applied in these diagonal elements, belonging to the contact layer, which met the following condition:

$$\varepsilon_i^0 (\varepsilon_i - \varepsilon_i^0) \geq 0 \quad (12)$$

meaning that the elastic energy was accumulated in an adaptive way (without supplying the energy to the system – the distortion must not exceed the strain). Figure 7.13 shows the history of the strain for an element belonging to the upper beam (sensor). The vibration was successfully damped after applying the control action four times ($\alpha = 4$).

7.4 Experimental Verification

7.4.1 Experimental Set-up

In order to validate the efficiency of the PAR strategy in vibration mitigation, a cantilever two-layered beam has been chosen as a demonstration. The experimental beam was submitted to free vibrations. The idea of the experiment was to control the delamination between layers. As described in the numerical model section (Section 7.3.2.1), the time during which the layers are disconnected should be short as compared to the full oscillation period. Results were compared with a reference case in which the layers were pressed together with a maximum force.

The experimental set-up consists of two adjacent aluminum bars clamped at the root. Bars are not connected with each other along the length, except at the tip, where they can be pressed together with the actuator device. The actuator used for the experiment consists of piezo-stacks and a mechanical displacement amplifier [11]. The resulting stroke can reach 0.23 mm and the maximum force is about 1300 N. Using this device, the layers can be pressed together with enough force to hold them together during the deformation process.

The actuator state is updated, based on a piezoelectric sensor reading placed close to the cantilever root. A general view of the set-up is shown in Figure 7.14.

7.4.2 Control Procedure

The process of adaptation of the system during free vibrations is based on a closed loop between the strain sensor reading and the voltage applied to the piezo-stacks of the actuator device. The process starts with the maximum voltage applied to piezo-stacks, i.e. with maximum force holding the layers together. The control algorithm recognizes a displacement peak at which the layers are disconnected for a time interval of 100 ms. The time interval at which the force is completely removed is significantly smaller in practice due to the internal constraints of the amplifier. In fact, the 100 ms interval was possibly the smallest at which the actual force could decrease to zero. The choice of the amplifier was a compromise, where on the one hand it was possibly the fastest reaction time and on the other hand it was the safety of piezo-stacks, which could be destroyed if too high current peaks were applied. After this, the layers were connected again until the next displacement peak is recognized; cf. the numerical simulations section (Section 7.3.2) for the expected behavior of the layers during the activation control. The control loop was updated with the sampling frequency of 100 Hz.

Figure 7.14 Experimental set-up: detailed view of the sensor and actuator

7.4.3 Results

7.4.3.1 Initial Tests

In the initial stage the system was submitted to free vibrations without applying any control procedure. At first no voltage was applied to the piezo-stacks, and then the maximum voltage was applied. The fundamental natural frequency of both systems was identified. The resulting Fast Fourier Transform (FFT) spectra are shown in Figure 7.15. A 22 % change in the fundamental frequency of the structure can be observed, which means that the actuator state can significantly influence the response of the system.

7.4.3.2 Semi-active Control

Free oscillations were introduced, with the initial displacement of 10 cm applied at the tip of the cantilever. After a few oscillations the control procedure was enabled. Then, after the delamination had been allowed in a few displacement extremes (in this particular case, in three consecutive peaks), the control procedure was disabled again. For the reference case the control procedure was disabled during the entire process. Disabling of the control procedure means that a constant maximum voltage is applied to piezo-stacks regardless of the sensor reading. Results are shown in Figure 7.16. Triggering the delamination in three consecutive peaks causes the vibration amplitude to be mitigated by over 80 %. The remaining control triggering occurs when the vibration is stopped almost completely and is of no importance. It can be seen that only a threefold delamination has a substantial effect on the structural vibrations. The remaining amplitude could be further mitigated if the control had been activated for longer periods of time.

It has to be emphasized that the actual delamination time is much shorter than the assumed 100 ms due the amplifier constraints. This means that the delamination, or ‘weakening’ of the structure, is allowed during only a few percent of the total oscillation period.

Figure 7.15 First natural frequency shift between the disconnected and connected layers

The system under investigation has quite high structural damping, which can be seen in Figure 7.16. Figure 7.17, in turn, illustrates the relative difference in amplitudes of the first 10 vibration peaks, as related to the reference case.

In order to release possibly most of the strain energy, the delamination has to coincide with the deformation peaks. Therefore the PAR efficiency is very sensitive to the time instant of the delamination. If it occurs too long before or after the maximum deflection, then the

Figure 7.16 Time history response of the systems

Figure 7.17 Relative difference in amplitudes as related to the reference case

Figure 7.18 Weak effect due to a wrong delamination trigger time

obtained response mitigation is less significant. If, in the worst case, the delamination occurs around the equilibrium position of the cantilever, then there is no effect at all since there are no deformation incompatibilities between the layers that could be 'frozen'. Figure 7.18 shows no damping effect due to a wrong delamination trigger time.

7.5 Possible Applications

Among possible applications are:

- pedestrian bridges,
- pipelines,
- truss structures,
- small-scale mechanical systems.

The two-layered beam discussed in Section 7.2 has the span and stiffness of an experimental, lightweight pedestrian bridge located in EMPA Laboratory, Switzerland. Cables sustaining the span, which completely change the dynamic behavior of the structure, were not modeled in the numerical example. In order to apply the PAR strategy effectively to this type of structure, mitigation of higher modes has to be accounted for. For this purpose, only one active member at the support is not sufficient. The detachable spring which was discussed in Section 7.3 could be used at the supports of pipeline systems in order to accumulate the deformation energy and convert it to kinetic energy, which can then be dissipated.

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